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2001 J. Phys.: Condens. Matter 13 2799

(http://iopscience.iop.org/0953-8984/13/12/304)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **13** (2001) 2799–2805 www.iop.org/Journals/cm PII: S0953-8984(01)18845-4

Energy exchange in 2D electron–electron collisions with dynamic screening

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Received 6 November 2000, in final form 29 January 2001

Abstract

The rapidity of thermalization processes within an excited electron gas is determined by the rate at which substantial energy exchanges can occur. To calculate the rate of such processes, as distinct from calculating the rate of scattering (which is the rate usually calculated), it is necessary to take into account the dynamic nature of screening. It is shown that this can be carried out analytically for the case of a non-degenerate electron gas in a 2D quantum well within the random-phase approximation. The results indicate resonant screening and anti-screening effects associated with the response of the polar lattice in a semiconductor. These effects are illustrated in the case of GaN at 77 K.

The rate at which an excited electron gas thermalizes is an important component determining the speed of response of electronic and optoelectronic devices, and one that has to be taken into account in interpreting phenomena associated with fast photoexcitation. A large amount of experimental and theoretical work has been carried out over the last 20 years, mainly on bulk GaAs and GaAs/AlGaAs quantum wells [1, 2]. Experiments have included light-scattering and coherent techniques that measure dephasing times, and time-resolved measurements of photoluminescence or absorption that measure energy-relaxation times. All experiments involve both electron and hole populations, so it is not always possible to disentangle electron–electron (e–e) rates from electron–hole (e–h), electron–plasma (e–pl) and electron–LO-phonon (e–ph) rates. Furthermore, these rates are density dependent and may therefore vary throughout an experiment. Dephasing times are observed to be some tens of femtoseconds whereas the observed energy-relaxation time for (e–ph) is of the order of hundreds of femtoseconds. Theoretical work has often assumed a static-screening model with rates computed numerically using Monte Carlo techniques [3–7]. Rates calculated this way are only for collisions involving small momentum and energy transfers. More recent studies on photoexcited populations have shown that the assumption of static screening underestimates the energy-relaxation rate and that dynamic screening should be taken into account [8–11]. The effect on the dephasing time in photoexcited populations can be to reduce the time by a factor of 2 [10] and to explain the behaviour of the exciton energy at strong excitation [11]. The effects of dynamic screening have

0953-8984/01/122799+07$30.00

also been studied for the case of electron–electron interactions involving plasmon emission in quantum wells [12, 13].

The intensive numerical nature of these studies, however necessary to relate a complex situation to what is observed in real experiments, tends to obscure the physics of the processes that are involved. It is the aim here to focus on a simpler system than is usually treated. Thus, it is assumed that only one type of carrier is involved in low concentration, and it is assumed that plasmon interactions can be ignored. We therefore can present an analytical model of energy relaxation via e–e scattering in a non-degenerate gas, so that the twin rôles of dynamic lattice screening and dynamic electron screening can be clearly illustrated. It is shown that energy exchanges above the TO phonon energy are substantially anti-screened, and at all substantial energy exchanges the rate is significantly greater than that predicted by theory that assumes static screening.

A fundamental assumption, adopted here, is that e–e collisions are essentially two-body processes. This assumption is valid as long as the screening length is less than the average distance apart of the electrons, and this imposes a lower limit to the density, which is of the order of 8×10^{10} cm^{-2} at room temperature, less at lower temperatures. (There is numerical evidence that the two-body model is inadequate at a density of 2×10^{10} cm^{-2} in GaAs at 300 K [14].) In a two-body collision an incident electron, wavevector $\boldsymbol{k}_1$, collides with a target electron, wavevector $\boldsymbol{k}_2$, and after collision the electrons have wavevectors $\boldsymbol{k}_1'$ and $\boldsymbol{k}_2'$. The frequency of this process is calculated in the Born approximation. Usually the calculation proceeds by first summing over the final states and then summing over the target states weighted by the probability of occupancy. This procedure calculates the overall scattering rate. However, in many cases the physically meaningful rates are momentum and energy exchange rates, which means that the order of summation must be such that the final sum should be over one of the final states suitably weighted by the amount of energy or momentum exchanged. The following calculation is of the rate of energy exchange.

In order to highlight the effects of dynamic screening, exchange and interference effects will be ignored and only intrasubband transitions will be considered. In the Born approximation the scattering rate for the process $\boldsymbol{k}_1 \rightarrow \boldsymbol{k}_1'$ in an isotropic, Maxwellian distribution is

$$W(\boldsymbol{k}_1, \boldsymbol{k}_1') = \frac{e^4 n}{8\pi\hbar A N_d} \int \mathrm{e}^{-E_2/k_B T_e} \frac{F^2(q)}{\varepsilon(\boldsymbol{q}, \omega)^2 q^2} \delta(E_1' + E_2' - E_1 - E_2) 2\,\mathrm{d}\boldsymbol{k}_2 \tag{1}$$

where n is the areal density of electrons, N_d is the effective density of states in the lowest subband, A is the area, $F(q)$ is the form-factor, $\hbar\boldsymbol{q}$ is the momentum transfer and $\varepsilon(\boldsymbol{q}, \omega)$ is the permittivity. Integration over $\boldsymbol{k}_2$ is straightforward. The integral over the angle between $\boldsymbol{k}_1$ and $\boldsymbol{k}_1'$ can be expressed in terms of a new variable:

$$u = \frac{q}{(k_1^2 - k_1'^2)^{1/2}} \tag{2}$$

so, after some manipulation and including the case for $k_1 < k_1'$, we obtain

$$W(k_1, k_1') = W_0 \int_{\gamma}^{\gamma^{-1}} \frac{F^2(u)}{|\varpi|^{3/2} \{\varepsilon(u, \varpi)/\varepsilon_\infty\}^2} \mathrm{e}^{\varpi/2} \frac{\exp\{-(|\varpi|/4)(u^2 + 1/u^2)\}}{u^2 \{(u^2 - \gamma^2)(1/\gamma^2 - u^2)\}^{1/2}} \,\mathrm{d}u$$

$$W_0 = \frac{e^4 n \hbar}{8\pi^{1/2} \varepsilon_\infty^2 m^* (k_B T_e)^2 A} \qquad \gamma = \left| \frac{k_1 - k_1'}{k_1 + k_l'} \right|^{1/2} \tag{3}$$

where ε_∞ is the high-frequency permittivity of the lattice and $\varpi = (E_1 - E_1')/k_B T_e$ is the normalized exchange energy. Equation (3) is just that derived by Esipov and Levinson [15], but with the form factor and screening factor included. The energy-relaxation rate for the

incident electron is then

$$Q = \int (E_1 - E_1')W(k_1, k_1')k_1' \,\mathrm{d}k_1' A/2\pi. \tag{4}$$

Dynamic effects enter screening via the factor $\boldsymbol{q} \cdot \boldsymbol{v}_{cm}$, where $\boldsymbol{v}_{cm}$ is the velocity of the centre of mass [16]. It is straightforward to show that this factor is nothing but $(E_1 - E_1')/\hbar$, which is the frequency associated with the energy loss by the incident electron. This follows from the conservation of crystal momentum and energy. Thus, for intrasubband transitions:

$$\begin{aligned} \hbar\omega &= \frac{\hbar^2}{2m^*}(\boldsymbol{k}_1 + \boldsymbol{k}_2) \cdot \boldsymbol{q} = \frac{\hbar^2}{4m^*}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_1' + \boldsymbol{k}_2') \cdot \boldsymbol{q} \\ \boldsymbol{q} &= \tfrac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_1' + \boldsymbol{k}_2' - \boldsymbol{k}_2) \end{aligned} \tag{5}$$

therefore

$$\hbar\omega = \frac{\hbar^2}{2m^*}(k_1^2 - k_1'^2).$$

The permittivity is composed of the sum of lattice and electronic contributions. In a polar semiconductor the lattice contribution in the long-wavelength limit is

$$\varepsilon_L(0, \omega) = \varepsilon_\infty \frac{\omega^2 - \omega_{LO}^2 + \mathrm{i}\omega\Gamma}{\omega^2 - \omega_{TO}^2 + \mathrm{i}\omega\Gamma} \tag{6}$$

where Γ is the decay rate. The electronic contribution can be obtained in the random-phase approximation neglecting the effects of exchange and correlation. A closed expression has recently been obtained by Lee and Galbraith [12]. The real part is

$$\begin{aligned} \varepsilon_{eR} &= \frac{e^2 m^* n F(q)}{2\pi\hbar^2 N_d q^2}\left[A_+\Phi\left(1, \frac{3}{2}, -\frac{\hbar^2 A_+^2}{2m^* k_B T_e}\right) + A_-\Phi\left(1, \frac{3}{2}, -\frac{\hbar^2 A_-^2}{2m^* k_B T_e}\right)\right] \\ A_\pm &= \frac{1}{2q}\left(q^2 \pm \frac{2m^*}{\hbar^2}\hbar\omega\right) \end{aligned} \tag{7}$$

where $N_d = m^* k_B T/\pi\hbar^2$ is the 2D density of states and $\Phi(1, 3/2, -z)$ is a confluent hypergeometric function, and the imaginary part is

$$\varepsilon_{eI} = \frac{e^2 m^* n F(q)}{2\pi\hbar^2 N_d q^2}\sqrt{\frac{\pi m^* k_B T_e}{2\hbar^2}}\left(\exp\left\{-\frac{\hbar^2 A_-^2}{2m^* k_B T}\right\} - \exp\left\{-\frac{\hbar^2 A_+^2}{2m^* k_B T_e}\right\}\right). \tag{8}$$

The quantity that appears in the expression for the rate is the square modulus:

$$\varepsilon(q, \omega)^2 = (\varepsilon_{LR} + \varepsilon_{eR})^2 + (\varepsilon_{LI} + \varepsilon_{eI})^2. \tag{9}$$

The rate for quasi-elastic scattering ($|\varpi| \ll 1$), the usual one calculated, cannot contribute much to the energy relaxation in the presence of static screening. The more relevant rate is that for substantial inelastic scattering ($|\varpi| \gg 1$). In this case the exponential in equation (3) will favour contributions to the integral from $u \approx 1$. Putting $u = 1$ in $F(q)$ and in $\varepsilon(q, \varpi)$ determines q in terms of ϖ via equation (2) and we obtain for a superthermal electron such that E, E' and $|E - E'| \gg k_B T_e$

$$\begin{aligned} W(k_1, k_1') = W_0 &\frac{F^2(u = 1)\,\mathrm{e}^{(\varpi/2 - |\varpi|/2)}}{|\varpi|^{3/2}\{\varepsilon(u = 1, \varpi)/\varepsilon_\infty\}^2(\gamma^{-1} - \gamma)}\frac{\pi^{1/2}}{2} \\ &\times[\mathrm{erf}[\varpi^{1/2}(\gamma^{-1} - 1)] + \mathrm{erf}[\varpi^{1/2}(1 - \gamma)]] \end{aligned} \tag{10}$$

so for the downward transition ($\varpi > 0$)

$$W(k_1, k_1') \approx W_0 \frac{F^2(u = 1)}{|\varpi|^{3/2}\{\varepsilon(u = 1, \varpi)/\varepsilon_\infty\}^2(E_1'/k_B T_e)^{1/2}}\frac{\pi^{1/2}}{2}$$

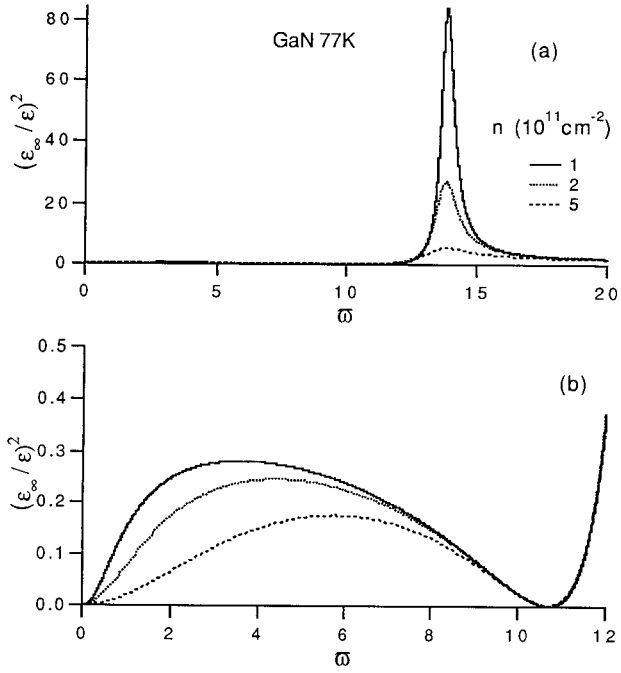

Figure 1. Screening function for GaN at 77 K. (a) Whole range; (b) low-energy range.

and for the upward transition ($\varpi < 0$)

$$W(k_1, k_1') \approx W_0 \frac{F^2(u=1)\,\mathrm{e}^{\varpi}}{|\varpi|^{3/2}\{\varepsilon(u=1,\varpi)/\varepsilon_\infty\}^2 (E_1/k_B T_e)^{1/2}} \frac{\pi^{1/2}}{2}. \tag{11}$$

Downward transitions are heavily favoured. The arguments of the hypergeometric functions can be written

$$\frac{\hbar^2 A_\pm^2}{2m^* k_B T} = \frac{\varpi}{4}\left(u \pm \frac{1}{u}\right)^2 \text{ and so } \frac{\hbar^2 A_+^2}{2m^* k_B T} \approx \varpi \text{ and } \frac{\hbar^2 A_-^2}{2m^* k_B T} \approx 0 \tag{12}$$

and the confluent hypergeometric can be approximated by $\Phi(1, 3/2, -\varpi) \approx 1/2\varpi$ with less than 17% error for $\varpi > 5$. With these approximations the integral in equation (4) for the dynamically screened energy-relaxation rate can be simply evaluated numerically. In order to avoid unnecessary complications involving well-width, we consider only the 2D case, though there is no intrinsic difficulty in allowing for the variation of the form factor with energy. It may be noted that the energy-relaxation time, τ_E, can be obtained from Q via $\tau_E = E/Q$, where E is the energy of the incident electron, and that it is also effectively the momentum-relaxation time when the energy exchanged is large, as it is in our calculation.

Neglecting screening altogether (apart from lattice static screening) we return to the energy-relaxation rate given for the 2D case by Esipov and Levinson [15], namely

$$Q = \frac{e^4 n}{32\varepsilon_s^2 \hbar}. \tag{13}$$

Comparison with the dynamic-screening result can be made by noting that for $n = 1, 2$ and 5×10^{11} cm^{-2}, $Q = 0.192, 0.383$ and 0.960 eV ps^{-1} respectively, with $\varepsilon_s = 9.0\varepsilon_0$. For energy losses greater than the LO energy comparison should be made with the results when the static

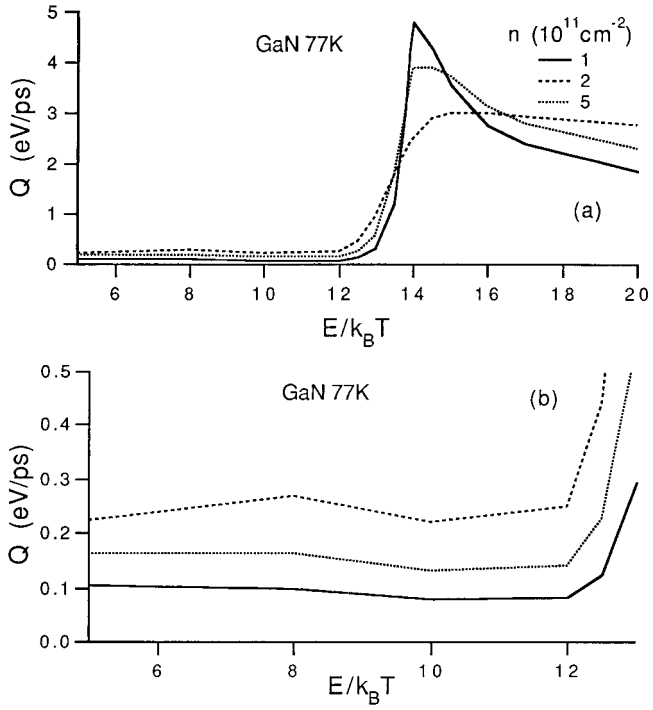

Figure 2. Dependence of energy-loss rate on initial energy. (a) Whole range; (b) low-energy regime.

permittivity in equation (13) is replaced by the high-frequency permittivity, $\varepsilon_\infty = 5.35\varepsilon_0$ i.e. $Q = 0.543$, 1.086 and 2.715 eV ps^{-1}. It is convenient to refer to these results as those for the unscreened case (even though there is the usual lattice dielectric response).

We illustrate the remarkable effects of dynamic screening for the case of GaN at 77 K in the 2D limit. Figure 1 depicts the reciprocal of the square modulus of the permittivity as a function of the energy exchange with $\hbar\omega_{LO} = 92$ meV, $\hbar\omega_{TO} = 71$ meV, $\hbar\Gamma = 1$ meV, $m^* = 0.23m_0$. (These values correspond to the cubic approximation for the wurtzite form, which has been shown to be valid [17].) There is a striking anti-screening effect for energy losses at and above the LO phonon energy whereas for energy losses below the TO phonon energy screening is positive and at the TO energy it is total. The resonance at the phonon energy is a consequence of the vanishing of the real part of the permittivity. Figure 2 shows the dependence of the energy-loss rate on initial energy. It is notable that the variation with density is relatively small, reflecting the fact that, away from resonance, the increased probability of scattering with increasing density is countered by the increase in screening. The resonance is limited essentially by the imaginary part of the electronic component of the permittivity. The latter weakens with decreasing density and consequently the resonance becomes more marked at lower densities. Figure 3 depicts the much smaller rates obtained assuming static screening.

The dynamic response of the polar lattice produces an order-of-magnitude difference in the energy-relaxation rates for electrons via e–e scattering above or below the optical-phonon band. Below the optical-phonon band the screened e–e interaction is still strong enough to determine an electron temperature, since the competition is only the quasi-elastic scattering of acoustic phonons. Above the optical-phonon band it has to compete with the emission of an

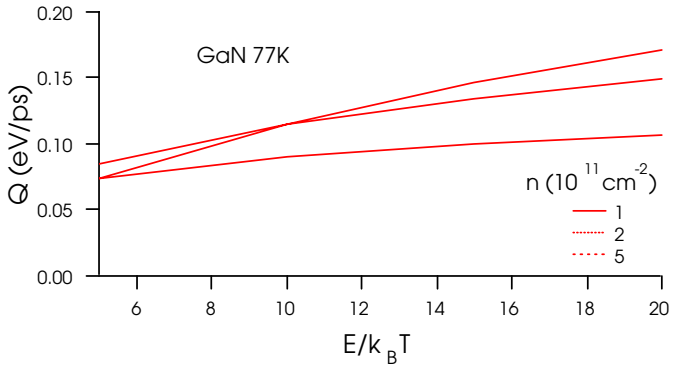

Figure 3. Dependence of energy-loss rate on initial energy assuming static screening.

LO phonon which, in GaN, has a characteristic time-constant of 10^{14} s^{-1}, corresponding to an energy-relaxation rate of about 9 eV ps^{-1}. The e–e rate is less than this, typically by a factor of two or more, so it is not possible to assume that an electron temperature is a viable concept in this range of energies.

As regards the dependence on electron temperature, the real part of the dynamic dielectric response is temperature independent (if the temperature dependance of the phonon energies and lifetime are ignored) and so the energy-loss rate in the low-energy regime is energy independent also. The imaginary part, however, reduces with increasing temperature, so it enhances the resonance with increasing temperature.

This analysis cannot replace a full numerical treatment, but it is adequate to provide an approximate quantitative picture of the effect of dynamic screening involving the polar lattice and to emphasize the invalidity of assuming the unscreened model. It may be noticed that taking the square modulus of the permittivity conceals the fact that for frequencies lying between the TO and LO phonon frequencies the interaction between two electrons is attractive. Indeed, in the absence of losses (imaginary components vanish) the condition $\varepsilon = 0$ indicates a novel long-lived excitation involving the exchange of virtual phonons between two electrons.

A connection with experiment cannot be made without including the effects of hot phonons, coupled modes, piezoelectric and other scattering mechanisms. It will also be necessary to take into account the quasi-2D form factors and the possibility of inter-subband processes. Our intention here has been limited to pointing out the importance of the dynamic response of the polar lattice to the electron–electron interaction.

Acknowledgments

I would like to thank Nick Zakhleniuk for many discussions and I would like to acknowledge the support given by EPSRC and the US Office of Naval Research.

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